The Castrop IOL Formula

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The basic IOL power formula is quite old; to our knowledge, it was first described by Fyodorov [1] and by Gernet and Ostholt [2]. This paraxial vergence formula considers three refractive surfaces, a spectacle correction (or target refraction) located at d_{vertex} in front of the cornea, a thin lens cornea with P_{cornea}, and an intraocular lens implant with refractive power P_{IOL} located at ELP behind the cornea:

$$P_{IOL} = \frac{n_{vitreous}}{AL - ELP} - \frac{1}{\dfrac{1}{\dfrac{1}{\dfrac{1}{P_{spectacle}} - d_{vertex}} + P_{cornea}} - \dfrac{ELP}{n_{aqueous}}}$$

All distances in [m].

$n_{cornea} = 1.376$.

$n_{air} = 1.000$ (rounded).

$n_{aqueous} = 1.336$.

All classical Gaussian optics IOL formulae date back to this approach. Many derivates exist. They differ mostly in how "ELP" (effective lens position) is dealt with. We used this equation as the basis for our IOL calculation. In daily practice, it makes sense to solve the equation for $P_{spectacle}$ instead of P_{IOL}.

In recent years, many formulae have emerged that are neither published nor disclosed or documented. Some of them provide great results, and some provide less convincing results under specific conditions. We feel it is crucial to understand how the formula acts, and how it processes the input data. Therefore, we would like to document our own approach in detail.

In classical formulae, we identified four typical sources of error that can be cured quite easily.

1. Most conventional formulae consider the cornea as a thin lens model and use a fictitious refractive index of either 1.3375 or 1.332 to convert the mean front surface radius measured paracentrally to "corneal power" K. As this approach tends to overestimate the corneal power by 0.4 to 1.1 D, the IOL power is underestimated accordingly. To compensate for this,

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the fictitious lens position ELP is moved to a position located behind the biconvex lens. This will lead to the next problem. To avoid this, corneal power is calculated using a thick lens model and the measured radii as "equivalent power" (distances in mm). If no data on the posterior curvature is available, we assume a

ratio of 0.84 that was derived from very large ssOCT data sets and which is very close to the accepted Liou & Brennan ratio [3]

$$r_{post} = 0.84 \cdot r_{ant}$$

for an untreated cornea. To avoid confusion with the traditional "K," we will call this P_{cornea}.

$$P_{cornea} = \frac{n_{cornea} - n_{air}}{r_{ant}} + \frac{n_{aqueous} - n_{cornea}}{r_{post}} - \frac{CCT}{n_{cornea}} \cdot \frac{n_{cornea} - n_{air}}{r_{ant}} \cdot \frac{n_{aqueous} - n_{cornea}}{r_{post}}$$

P_{cornea} is referenced to the principal plane. From a physical point of view, it would be correct to reference the front vertex. As we want to keep "compatibility" with manufacturer's indications like Tomey's ACCP or Heidelberg's TCRP, we kept the principal plane as a reference. This will become important when it comes to odd cornea, e.g., post LASIK when we can simply replace the whole P_{cornea} term with a power value derived by the manufacturer's software. The difference between the principal plane and front vertex will be ≈ 50 µm in an average cornea, so the systematic deviation will be quite small.

2. As the corneal power is overestimated, a given lens power with a realistic ELP (ELP matches the physical or anatomical position of the IOL) would lead to an underestimation of the lens power and therefore to a hyperopic error. When ELP is assumed to be located behind the physical IOL position, the resulting IOL power will increase, and the error is compensated on average. However, in eyes with unusual combinations of axial length and corneal radii, this will lead to systematic errors. This can be avoided when the ELP is very close to its real

position inside the eye. In all biconvex IOL designs, the principal plane of the IOL will be located between the front and back IOL vertex, and in most IOL models on the market, the position is close to the center plane of the lens (see below). A very simple equation according to Olsen [4] had been used in an early version of the Castrop formula (distances in mm):

$$ELP = -0.18 + ACD + C \cdot LT$$

"C" describes the fraction of crystalline lens thickness where the ELP will be presumed. It can vary with haptic and optic design as well as step vault at the edge. Typical values will be between 0.35 and 0.42.

However, IOL position prediction can be further improved when axial length and corneal radii are included in the regression. In contrast, corneal diameter does not reduce the variance in the prediction significantly; therefore, it was omitted. The following equations were derived from a large set of eyes where crystalline lens thickness and position and IOL position were measured optically (distances in mm).

$$ELP = 0.610 + 0.049 \cdot AL + 0.729 \cdot CCT + 0.680 \cdot AQD - 0.123 \cdot r1_{ant} + C \cdot LT + H$$

(historical version used in Wendelstein's paper)

$$ELP = 0.045 \cdot AL + 0.761 \cdot ACD - 0.042 \cdot \overline{r}_{ant} + C \cdot LT + H$$

(recent version)

where r1$_{ant}$ (flat r) as well as $\overline{r}_{ant}$ refers to the base curve of the corneal front surface. In eyes with prior corneal refractive surgery or corneal pathology, corneal radii should be left out and the following equation without corneal radii used instead.

$$ELP = -0.09 + 0.037 \cdot AL + 0.602 \cdot CCT + 0.715 \cdot AQD + C \cdot LT + H$$

(historical version used in Wendelstein's paper)

$$ELP = 0.036 \cdot AL + 0.753 \cdot ACD + C \cdot LT + H$$

(recent version)

Alternate multiple regressions using AS-OCT data like lens diameter, lens curvatures, and lens equator position have also been successfully tested and can reduce prediction error even further. However, these data will not be available to most users and are, therefore, not included in the current version of the formula.

IOLs with planar haptics and steeper anterior radii will have a smaller C than IOLs with posteriorly angulated or stepped haptics and/or designs where the main power is located on the posterior curvature. It is important that "C" optimization does not yield a significant skewness (the median is significantly different from the arithmetic mean). The remaining small offsets can be compensated for by adding an offset to the presumed refraction ("R" for "Rauxel"). This avoids systematic errors. The version of the formula that was used in Wendelstein's paper [5] used only C and R and worked quite well. However, Langenbucher showed H as a designated offset to be beneficial, so it is recommended to use it in conjunction with the new ELP regressions from now on.

3. Axial length is measured optically. This means that the optical path length has to be converted into a *geometrical* path by dividing it by the refractive index. However, the refractive index of the eye is not constant. For the average eye, the group refractive index will be assumed as 1.3549 according to the first IOLMaster versions [6]. In very long eyes, the fraction of vitreous will be larger, and consequently, the group refractive index will be smaller leading to hyperopic error. The opposite is true for short eyes. To overcome this problem, the best solution would be to replace the group refractive index with a sum-of-segments approach with different indices for each segment of the eye instead. There will still be some imprecision as the index of the cataractous lens material is unknown, but systematic errors will be significantly reduced. Unfortunately, none of the biometers able to *measure* sum-of-segments will *indicate* "new" AL but use the "old" value instead (FDA, compatibility issues). We have to thank Cooke [7] for publishing a regression formula that provides a linear regression for a correction of the axial length derived from a LenStar LS900 biometer, which mimics the sum-of-segments.

4. We used Cooke's regression to transform traditional optical AL to "AL$_{new}$."

$$AL_{new} = 1.23854 + 0.95855 \cdot AL_{old} \\ -0.05467 \cdot LT$$

5. Some small systematic error will remain due to lens properties, and surgical and optometric technique and needs to be adjusted. In conventional formulae, several influencing variables are squeezed into the ELP (e.g., A constant). The most important ones will be the lane distance for refractometry, ambient light, haptic design, asphericity of surfaces (or spherical aberration of the pseudophakic eye), decentration (the more aspheric, the more hyperopic error), or tilt of the lens and capsulotomy properties. This will unavoidably lead to trend errors. In our opinion, it is

better to compensate for systematic refractive deviation by an additional simple offset instead of fudging the ELP.

It is important to understand that "ELP" in the context of the Castrop formula consists of two parts. First, the Lens Equator position (LEQ) will be derived from preoperative input variables (AL, ACD, LT, and [r]) by a multiple linear regression. This resembles an *anatomical* position that can actually be measured by an Anterior Segment OCT. In the future, deep learning algorithms may replace the multiple regression leading to even better results [8]. In this regression, "C" acts as a coefficient (see equations).

Depending on the IOL design, the relevant principal plane H′ will differ from the lens equator plane (LEQ). This is handled by an offset in the linear regression that we call "H" ("Homburg") as its use as a third degree of freedom was suggested by Langenbucher. In an equiconvex IOL, H′ will be located posterior to LEQ, in a typical modern IOL with a steeper anterior curvature, H′ will move anteriorly. When the exact design data of the IOL is known for all power values (Coddington factor), H could be adjusted systematically for any power step. In daily practice, a single "H" would represent the IOL model. It is well known that discrete steps in shape factors (e.g., Alcon SA60AT 25.0–29.5) may lead to systematic deviations.

We feel that every surgeon should do subsequent work on his refractive outcomes. We also think it is more appropriate to add a third constant (besides C and H) instead of fudging the ELP. We call this constant "R" for "Rauxel." In recent studies, we defined "emmetropia" for a lane distance of 6 m = 20 ft. If emmetropia shall be defined for infinity, R should be changed

accordingly (decrease R by $\frac{1}{6}$).

At the moment, optimization of 2 formula constants (C and R) or 3 formula constants (C, H, and R) is performed sequentially starting with C and H based on a multivariable linear regression and in a second step by adjusting R to nullify the mean signed formula prediction error for a set of

clinical data. Langenbucher developed an algorithm to optimize all three degrees of freedom simultaneously using nonlinear optimization strategies. With a Levenberg–Marquardt algorithm [9], all 3 constants of the Castrop formula can be optimized *en bloc* for minimization of root mean squared refraction error as the target criterion. This will soon be integrated into the IOLcon.org Website.

To summarize, the core of our formula is identical to the basic IOL power formula. Corneal power will be derived from radii using thick-lens Gaussian optics; if posterior radii and/or CCT are not available, they will be modeled according to Liou & Brennan. ELP is predicted from a multiple regression developed from true anatomical data enhancing Olsen's C concept ("Castrop" constant with or without "Homburg" offset). If the cornea has been tampered with or is difficult to measure, a simpler regression omitting corneal radius is recommended. Axial length is transformed according to Cooke. The remaining systematic offsets are accounted for by adding an offset R ("Rauxel").

The formula attempts to eliminate systematic errors (axial length, cornea, and chamber depth) as much as reasonable. It can also be used in post-LASIK eyes with great success if the "true corneal power" can be measured and calculated separately, e.g., CASIA2 ACCP or Anterion TCRP. The derived corneal power can be used to overwrite P_{cornea}. Alternatively, P_{cornea} can also be used, but it must be kept in mind that our simple Gaussian formula cannot deal with aspheric surfaces appropriately. It can be used in minus power cases as well as IOL powers up to and even beyond 40 D without further adjustments and good precision [5].

However, the limits of Gaussian optics still apply. Asphericity, decentration, and tilt of optical elements cannot be dealt with directly. The finer details of the Liou and Brennan eye model cannot be used to the fullest advantage. To break the chains, Gaussian optics would have to be replaced by geometric optics ("raytracing"). Unfortunately, as the design properties of the specific IOLs are not disclosed, and the local corneal power and height data might be unreliable
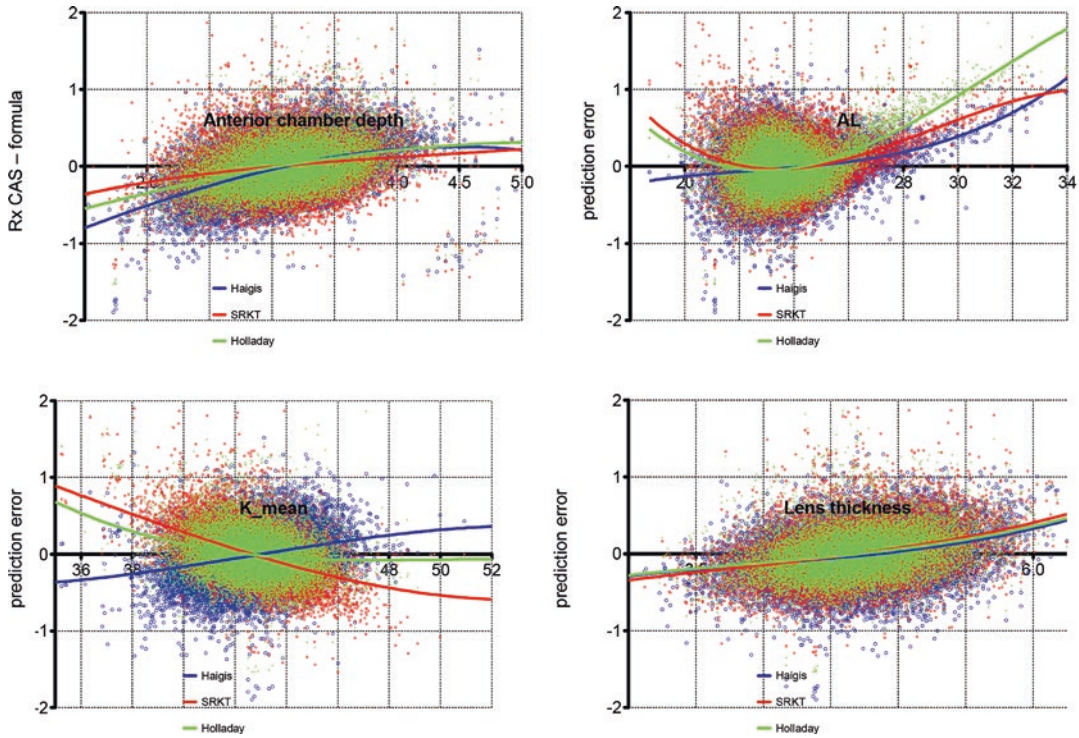
Fig. 38.1 Trend errors of Haigis, Holladay, and SRK/T with ACD, axial length, corneal power, and crystalline lens thickness

due to limitations in the respective measurement techniques such as anterior segment OCT or even more Scheimpflug imaging, this is not applicable in clinical routine, and the Gaussian approach would still make sense.

We believe that trend errors immanent to classical IOL formulae ([10–12] should be avoided whenever possible. This would specifically improve IOL calculation in any eye that is far away from statistical "normality," the especially short axis in combination with flat radii. It would also get you rid of the ritual of choosing from different formulae depending on the biometry data. As mentioned above, the way to achieve this is to avoid skewed outdated eye models and mixing up properties that do not belong together.

In a data set of 904 consecutive eyes, the Castrop formula achieved a standard deviation of the prediction error of 0.35 D (mean absolute error MAE = 0.28 D), compared to 0.39–0.42 D (MAE 0.31–0.34 D) for the classical formulae

Haigis, Hoffer Q, Holladay, and SRK/T – still very good values when compared to the literature (Fig. 38.1). Threefold optimization with the new ELP regression will improve results slightly but will yield more robust results for certain IOL such as B&L EnVista MX60.

When spherical IOLs are excluded (n = 365), the standard deviation will decrease to 0.31 D for the Castrop formula and 0.37–0.41 D for the classical formulae. In aspheric IOL, refraction will be more precise due to less pseudo accommodation; hence, the relative difference between formulae will increase as one of the major sources of stochastic error is decreased.

In very short eyes, the relative advantage will be even greater. The performance in these difficult eyes will be better than any classical formula and on par with Okulix raytracing, Pearl DGS, and Kane formula [5]. It compares very favorably to other modern formulae in normal and short eyes [13–15] (Fig. 38.2).

Fig. 38.2 Prediction error as box plot for 13 classical and new formulae. This is a data set of 95 very short eyes implanted with IOL powers of 30 D or more. This can be directly compared to [13]. The box plot was chosen to visualize the mean as well as the standard deviation/spread
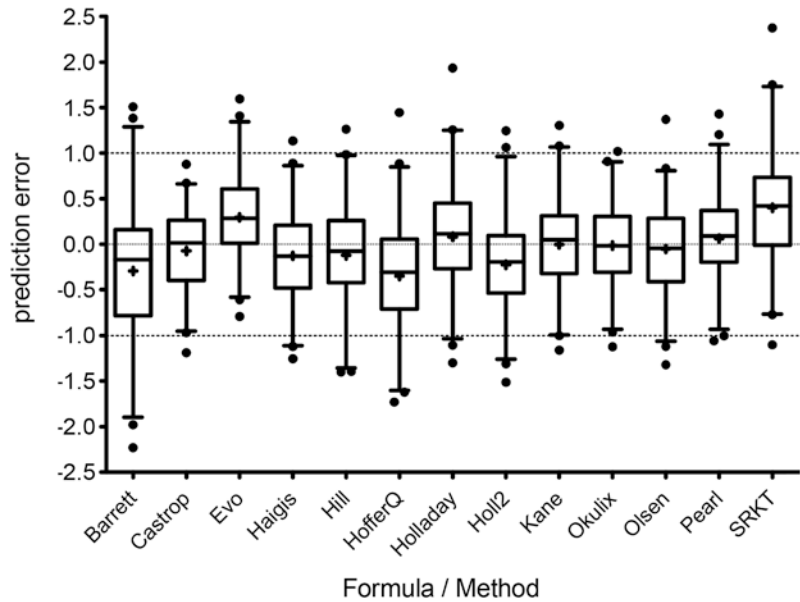
The formula is available as an Excel spreadsheet. The screenshot will give you an impression (Fig. 38.3). Optimized constants for six different acrylic IOLs used in our clinic have been derived, see Table below (Tables 38.1 and 38.2). As more postoperative data are coming in, 2- or 3-way optimization can be carried out fully automatically using Langenbucher's software. We now recommend using the web-based version that includes a batch processing option: https://iolcon.org/lpcm.php

A stand-alone version in executable code will soon be available, see Screenshot (Fig. 38.4). This software tool is capable of calculating toric (or stigmatic) intraocular lenses for any spherocylindrical target refraction as well as predicting spherocylindrical refraction at the spectacle plane. For calculation of the IOL power, the following parameters are required: target refraction, vertex distance, flat and steep front surface radii, central corneal thickness (optional), flat and steep corneal back surface radii (optional), phakic anterior chamber depth and lens thickness, axial length, as well as formula constants C, H, and R. For prediction of the postoperative spherocylindrical refraction, the following parameters are required: equivalent power and torus (optional) of the lens implant, vertex distance for spectacle correction, flat and steep front surface radii, central corneal thickness (optional), flat and steep corneal back surface radii (optional), phakic anterior chamber depth and lens thickness, axial length, as well as formula constants C, H, and R. In addition, this software is able to batch-process data from an Excel table if available in a special template format ("Browse"). In this batch processing lens, power is calculated, refraction is predicted en bloc from a data set, and the respective results are added with new columns in the Excel sheet. With a sufficient number of data, the Castrop formula constants are derived using nonlinear optimization for the root mean squared prediction error of equivalent refraction. These optimized constant data are also added to the excel table. A web-based version of the formula is available at https://iolcon.org/lpcm.php

It is our concern that every detail of this IOL calculation approach is transparent and public domain. We believe this is the best way to guarantee scientific integrity and improve clinical outcomes without barriers or paywalls. Further improvements in measuring hardware can easily be adopted. It does make sense to merge the formula with an IOL database like IOLcon.org as all other modern formulae are not disclosed and can, therefore, not be optimized with user-generated data in an automated way.

Fig. 38.3 Screenshot of Excel spreadsheet used for IOL studies. It is self-explaining and can be used to process large data sets for clinical studies

Table 38.1 Optimized constants C and R as used in the BJO paper (old ELP regression)

IOL	Optic	Haptic	C	R	Sample size
Alcon Clareon	Aspheric	1 piece planar C	0.40	0.24	40
Alcon Acrysof SA60AT/SN60AT	Spheric	1 piece planar C	0.37	0.00	296
B&L EnVista MX60	Aspheric neutral	1 piece planar C	0.42	−0.02	243
Hoya Vivinex	Aspheric	1 piece planar C	0.40	0.14	30
J&J AAB00	Spheric	1 piece stepped C	0.39	−0.15	85
J&J ZCB00	Aspheric	1 piece stepped C	0.41	0.24	91

Table 38.2 Optimized with two/three constants C, (H), and R using the new ELP regression. 3-way optimization for $n < 50$ is not sensible. Any result with $n < 50$ should be used with caution. In parenthesis: improvement of variance of 3-way vs. 2-way optimization

IOL	Optic	Haptic	C	H	R	Sample size
Alcon Acrysof SA60AT/ SN60AT	Spheric	1 piece planar C	0.34 0.34	0.00 0.01	0.07 0.05	296 (< 0.1%)
Alcon Clareon	Aspheric	1 piece planar C	0.35 –	0.00 –	0.40 –	40
B&L EnVista MX60	Aspheric neutral	1 piece planar C	0.41 0.36	0.00 0.50	−0.01 −0.45	243 (5.6%)
Hoya Vivinex	Aspheric	1 piece planar C	0.40	0.00	0.00	30
J&J AAB00	Spheric	1 piece stepped C	0.390 0.42	0.00 −0.20	−0.28 −0.22	85 (1.7%)
J&J ZCB00	Aspheric	1 piece stepped C	0.43 0.42	0.00 0.10	−0.07 −0.14	91 (0.8%)

Fig. 38.4 Screenshot of stand-alone software written by Langenbucher. It is possible to import spreadsheets for batch processing

References

1. Fyodorov SN, Kolinko AI. Estimation of optical power of the intraocular lens. Vestn Oftalmol. 1967;4:27.
2. Gernet H, Ostholt H, Werner H. Die präoperative Berechnung intraocularer Binkhorst-Linsen. In: 122. Versammlung des Vereins Rheinisch-Westfälischer Augenärzte. Balve: Zimmermann; 1970. p. 54–5.
3. Liou HL, Brennan NA. Anatomically accurate, finite model eye for optical modeling. J Opt Soc Am A Opt Image Sci Vis. 1997;14(8):1684–95.
4. Olsen T, Hoffmann PC. C constant: new concept for ray tracing-assisted intraocular lens power calculation. J Cataract Refract Surg. 2014;40(5):764–73.
5. Wendelstein J, Hoffmann PC, Hirnschall N, Fischinger IR, Mariacher S, Wingert T, et al. Project hyperopic power prediction: accuracy of 13 different concepts for intraocular lens calculation in short eyes. Br J Ophthalmol. 2021;106(6):795–801.
6. Haigis W, Lege B, Miller N, Schneider B. Comparison of immersion ultrasound biometry and partial coherence interferometry for intraocular lens calculation according to Haigis. Graefes Arch Clin Exp Ophthalmol. 2000;238(9):765–73.
7. Cooke DL, Cooke TL. Approximating sum-of-segments axial length from a traditional optical low-coherence reflectometry measurement. J Cataract Refract Surg. 2019;45(3):351–4.
8. Langenbucher A, Szentmáry N, Wendelstein J, Hoffmann PC. Artificial intelligence, machine learn-

ing and calculation of intraocular lens power. Klin Monatsbl Augenheilkd. 2020;237(12):1430–7.

9. Langenbucher A, Szentmáry N, Cayless A, Müller M, Eppig T, Schröder S, et al. IOL formula constants - strategies for optimization and defining standards for presenting data. Ophthalmic Res. 2021;64(6):1055–67.

10. Haigis W. IOL calculation according to Haigis [Internet]. 1996 [cited 2012 Jan 25]. http://www.augenklinik.uni-wuerzburg.de/uslab/ioltxt/haie.htm.

11. Holladay JT, Prager TC, Chandler TY, Musgrove KH, Lewis JW, Ruiz RS. A three-part system for refining intraocular lens power calculations. J Cataract Refract Surg. 1988;14(1):17–24.

12. Retzlaff JA, Sanders DR, Kraff MC. Development of the SRK/T intraocular lens power calculation formula. J Cataract Refract Surg. 1990;16:333–40. Errata: 1990;16:528 and 1993;19(5):444–446.

13. Kane JX, Melles RB. Intraocular lens formula comparison in axial hyperopia with a high-power intraocular lens of 30 or more diopters. J Cataract Refract Surg. 2020;46(9):1236–9.

14. Melles RB, Kane JX, Olsen T, Chang WJ. Update on intraocular lens calculation formulas. Ophthalmology. 2019;126(9):1334–5.

15. Hipólito-Fernandes D, Elisa Luís M, Gil P, Maduro V, Feijão J, Yeo TK, et al. VRF-G, a new intraocular lens power calculation formula: a 13-formulas comparison study. Clin Ophthalmol. 2020;14:4395–402.