56 Calculation of Phakic and Pseudophakic Additional Lenses

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Introduction to Phakic and Pseudophakic Additional Lenses

Traditional methods to compensate for refractive errors are eyeglasses and contact lenses. Especially when correcting higher ametropia, astigmatism, and oblique light incidence, spectacles can themselves create new aberrations, and the frequent use of contact lenses may lead to intolerance, mostly in combination with dry eye syndrome. To overcome these issues, surgical refractive procedures can offer permanent and convenient results. Compared to keratorefractive interventions with excimer and femtosecond laser technologies, additional (implantable) lenses have significant advantages: They allow for a wider range of applications, they do not intensify dry eye syndrome, and the procedure is reversible at any time. Even years after the primary implantation, an exchange or explantation of an additional lens is possible with little surgical effort and minimal risk to surrounding tissues [1]. Additional IOLs are placed anterior to the crystalline lens (phakic Add-on) or artificial intraocular lens (pseudophakic Add-on). Possible locations are within the anterior eye chamber (haptics at iridocorneal angle or at the front of the iris) or within the posterior chamber in the sulcus ciliaris (Fig. 56.1). Because of potential complications with anterior chamber lenses, these types of IOL are used relatively rarely today. Theoretically, the nodal points of the IOL and eye will become closer together with the increasingly posterior placement of the Add-on, reducing disturbing photic phenomena. Furthermore, the greater distance to the cornea helps to prevent endothelial cell loss. On the other hand, in the case of a phakic eye, such placement increases the risk of triggering cataract development.

In young refractive patients with a clear lens and sufficient accommodation, **phakic Add-on IOLs** are an alternative to keratorefractive procedures, especially in eyes with higher myopia. Despite potential risks of pigment dispersion, pupillary block, and cataract development, modern phakic IOLs are shown to be safe, effective, and stable in many studies [2–5]. The surgical skills for the implantation, exchange, or explantation of these IOL are similar to cataract surgery, and in contrast to corneal laser surgery, the necessary equipment is considerably less expensive. As with all intraocular procedures, there are associated general surgical risks. In addition, a

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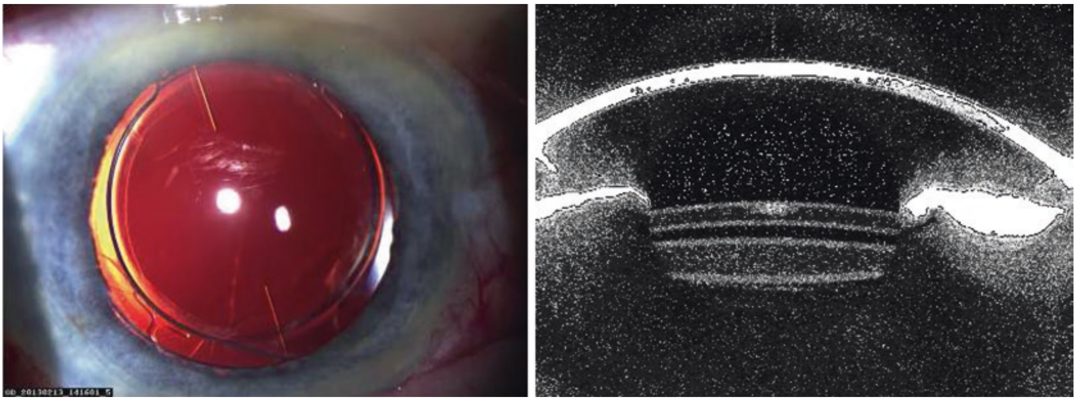

Fig. 56.1 Left: Toric Add-on IOL in front of the primary IOL in the capsular bag. Right: Scheimpflug image of the anterior eye segment with an even gap between the Add-on IOL in the sulcus ciliaris and the IOL in the capsular bag

progressive shallowing of the anterior chamber resulting from increasing lens thickness with age might contribute to the abovementioned issues. A pending issue of toric Add-on models is their potential rotational instability, which could induce crossed cylinders and a deteriorated visual performance even years after surgery [6, 7].

There are four main indications for pseudophakic Add-on IOLs:

1. Within the power calculation of IOLs before cataract or refractive lens surgery, all relevant parameters such as keratometric data, anterior chamber depth, lens thickness, and axial eye length can be measured by modern biometers with high precision. However, despite highly optimized measuring and power calculation methods, postoperative refractive surprises can still occur in some cases. Under such circumstances, pseudophakic Add-on IOLs are a welcome option for fine-tuning [8–13].
2. In situations where the patient decides on the option of pseudo-accommodation only after lens surgery, multifocal Add-ons provide a suitable alternative [8, 14–20] and a persisting deviation from emmetropia can be corrected at the same time.
3. The implantation of an IOL during congenital cataract surgery is another area of application. In the majority of cases, the eye of the child is still growing at the time of surgery and refractive conditions will change significantly. Because of the limited compliance of a small child, spectacles and contact lenses might not be an optimal solution and the exchange of the IOL in the bag is virtually impossible because of the massive ingrowth. For these reasons, exchangeable additional IOLs can be very helpful within the course of postoperative treatment [21].
4. A fourth indication for Add-ons is after keratoplasty, where the crystalline lens has been replaced before or during this operation [11]. In those cases, the prediction of the appropriate lens power may fail due to the unpredictable or varying corneal power. Here an Add-on IOL can be used to correct any persisting cylindrical and equivalent refraction error [22].

The fundamental **calculation strategy for additional IOLs** was described in 1988 by the so-called Van der Heijde formula [23]. In this paper, a spherical phakic Add-on IOL was calculated for a myopic correction using classical vergence transformation. Langenbucher et al. generalized this formula for the calculation of toric phakic IOLs with spherocylindrical target refraction using a vergence-based formalism by transferring the position of refractive correction from the spectacle plane to the IOL plane [24]. In contrast to intraocular lenses in the capsular bag, the calculation of an additional IOL is based on manifest subjective refraction, axial distance between spectacle back vertex and corneal front vertex, corneal curvature, and the axial position of the Add-on in the eye with respect to the corneal front vertex [22, 24].

Calculation of Additional Lenses

The special situation of calculating additional lenses relates to the transfer, in part or in full, of a preexisting refraction mostly at the spectacle plane, to the plane of the additional lens [2, 3, 15]. This means that there is no change in the optical system posterior to the additional lens plane, and therefore, we have to consider only the anterior eye segment for the calculation of the lens power or for the lateral magnification [24].

There are several options for calculating additional lenses: Calculation could be performed using linear Gaussian optics within the paraxial space, either with formulae based on vergence transform or with matrix algebras, or with ray-tracing strategies based on a ray bundle traced through all refractive surfaces and optical media from the object plane to the plane of the additional lens.

Figure 56.2 displays by way of example the optical model used for calculating the power of an Add-on, for the situation of a phakic lens implantation in the ciliary sulcus. In the upper graph, we have the preoperative situation with a spectacle correction at vertex distance (VD) in front of the cornea, and in the lower graph, the spectacle correction is transferred to the plane of the Add-on, which is located slightly in front of the crystalline lens.

The postoperative position of the Add-on (ELP) can be estimated from the position of the anterior surface of the crystalline lens (CRL for phakic lenses) or the replacement lens (IOL for pseudophakic lenses) and the vault. This vault corresponds to the interspace between the Add-on

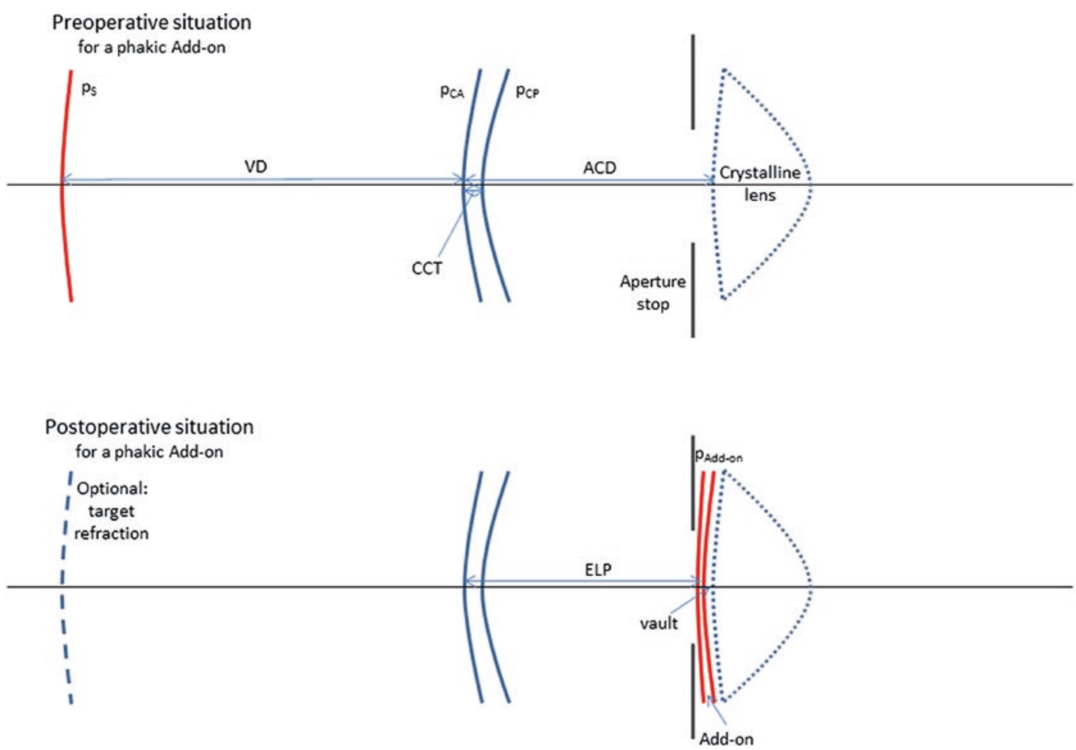

Fig. 56.2 Schematic drawing of the situation before (upper graph) and after (lower graph) implantation of an additional lens (Add-on). The axial position of the Add-on (ELP) is derived from the measured anterior chamber depth (ACD). The aperture stop of the optical system is assumed to be located at the Add-on plane

and the CRL or IOL and ranges between 0.2 and 0.5 mm. Therefore, a biometric measurement prior to Add-on implantation—preferably with optical biometry—is mandatory for the prediction of the estimated Add-on position ELP [24–27].

Our preferred calculation method for the Add-on is using matrix algebra, as the concept directly adds value by predicting the change of lateral magnification (ΔM). Using vergence transform formulae, the estimation of the magnification before and after Add-on implantation requires a separate calculation step. The strategy of matrix calculation is based on a system matrix, which describes and characterizes the paraxial optical properties of the relevant optical part of the eye [28–30]. This system matrix is composed of a product of refraction and translation matrices: A refraction matrix describes the change of ray direction as the ray passes through this surface, and a translation matrix describes the change in lateral ray position as the ray passes through a homogeneous optical medium. For stigmatic (non-toric) situations, the system matrix and all refraction and translation matrices are of dimension 2 × 2. With an incident ray defined in terms of its slope α_0 and height h_0, the slope α and height h of the ray exiting the optical system described by the system matrix S [28] are defined by

$$\begin{pmatrix} \alpha \\ h \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \alpha_0 \\ h_0 \end{pmatrix} = S \begin{pmatrix} \alpha_0 \\ h_0 \end{pmatrix}. \tag{56.1}$$

The system matrix S is derived from the product of the respective refraction and translation matrices considered in reverse order (against the ray direction). The refraction and translation matrices P and T are of the form

$$\begin{aligned} P &= \begin{pmatrix} 1 & -p \\ 0 & 1 \end{pmatrix} \\ T &= \begin{pmatrix} 1 & 0 \\ \frac{d}{n} & 1 \end{pmatrix}, \end{aligned} \tag{56.2}$$

where p refers to the surface power $p = (n' - n)/r$ (n' and n refer to the refractive indices behind and in front of the refractive surface of radius r), and d and n refer to the thickness of the refractive index of the homogeneous medium. Both situations of the anterior eye segment, from the spectacle plane to the Add-on plane before and after implantation of the Add-on, are described by system matrices S_{pre} and S_{post}. For example, for a thick lens model of the cornea, both matrices read

$$\begin{aligned} S_{pre} &= T_{iELP} P_{CP} T_{CCT} P_{CA} T_{VD} P_S \\ S_{post} &= P_{Add-on} T_{iELP} P_{CP} T_{CCT} P_{CA} T_{VD} \end{aligned}, \tag{56.3}$$

where T_{iELP}, T_{CCT}, and T_{VD} refer to the translation matrices for the aqueous depth, the cornea, and the vertex distance, and P_{Add-on}, P_{CP}, P_{CA}, and P_S refer to the refraction matrices for the Add-on, the corneal back and front surface, and the refraction correction before Add-on implantation, which is to be transferred (in this case fully) to the Add-on plane.

To obtain the same focus position, the exit vergences of S_{pre} and S_{post} at the Add-on plane must be identical. This means that with

$$S_0 = T_{iELP} P_{CP} T_{CCT} P_{CA} T_{VD} = \begin{pmatrix} A_0 & B_0 \\ C_0 & D_0 \end{pmatrix} \tag{56.4}$$

for an object located at −∞ (with an entrance vergence of 0 or slope angles α_0 '= 0) and a preoperative refraction at the spectacle plane of p_S, the following condition must be fulfilled:

$$\left(B_0 - A_0 p_S\right) D_0 = \left(D_0 - C_0 P_S\right)\left(B_0 - D_0 p_{Add-on}\right) \tag{56.5}$$

Reformulating Eq. (56.5) yields the refractive power of the Add-on (p_{Add-on}):

$$p_{Add-on} = \frac{B_0}{D_0} - \frac{\left(B_0 - A_0 p_S\right)}{\left(D_0 - C_0 p_S\right)}. \tag{56.6}$$

The lateral magnification before and after implantation of the Add-on is easily obtained from the respective system matrices S_{pre} and S_{post} [30, 31]. If an optical system S is corrected, either matrix element C or D in Eq. (56.1) equals zero (depending on whether it is corrected for far objects ($D = 0$) or for objects at finite distances ($C = 0$)). Here, our systems S_{pre} and S_{post} are not corrected; therefore, we have to select the chief ray [31] for evaluation of magnification properties. Assuming that the aperture of the system is located at the Add-on plane, this means $h = C\alpha_0 + Dh_0 = 0$, or $h_0 = -C/D\ \alpha_0$. Inserting this into Eq. (56.1) yields a relative lateral magnification $M_{pre/post}$ of

$$M_{pre/post} = \frac{\alpha}{\alpha_0} = A - B\frac{C}{D}, \qquad (56.7)$$

A relative change in lateral magnification of ΔM of M_{post}/M_{pre}.

Clinical Example 1

With a phakic lens, preexisting spectacle correction $p_S = -7$ dpt at a vertex distance VD = 12 mm to be transferred to a correction at ELP = 3.4 mm behind the corneal front apex (e.g., phakic anterior chamber depth: 3.6 mm, vault: 0.2 mm). With a corneal front/back surface radius of 7.77/6.4 mm, a central corneal thickness of 500 µm, and refractive indices of air/cornea/aqueous of 1.0/1.376/1.336, Eq. (56.4) for S_0 becomes

$$S_0 = \begin{pmatrix} 0.4953 & -42.2511 \\ 0.0132 & 0.8907 \end{pmatrix}$$

The power of the Add-on is derived from Eq. (56.6) as $p_{Add\text{-}on} = -7.9925$. Using Eq. (56.3) gives

$$S_{pre} = \begin{pmatrix} 0.4953 & -38.7843 \\ 0.0132 & 0.9833 \end{pmatrix}$$

$$S_{post} = \begin{pmatrix} 0.6010 & -35.1322 \\ 0.0132 & 0.8907 \end{pmatrix}$$

According to Eq. (56.7), we calculate a relative magnification of $M_{pre} = 1.0170$ and $M_{post} = 1.1227$ and an increase in lateral magnification of 10.4% ($\Delta M = 1.1040$).

Clinical Example 2

In this example, we consider a pseudophakic additional lens, with a post-cataract spectacle correction of $p_S = +3.5$ dpt at a vertex distance VD = 14 mm to be converted to a correction at an ELP = 4.8 mm behind the corneal front apex (e.g., pseudophakic anterior chamber depth: 5.1 mm, vault: 0.3 mm). Assuming a corneal front/back surface radius of 7.9/6.5 mm, a central corneal thickness of 550 µm, and refractive indices of air/cornea/aqueous of 1.0/1.376/1.336, Eq. (56.4) reads for S_0 becomes

$$S_0 = \begin{pmatrix} 0.4206 & -41.5582 \\ 0.0155 & 0.8488 \end{pmatrix}$$

The power of the Add-on is derived from Eq. (56.6) as $p_{Add\text{-}on} = 5.1894$. Using Eq. (56.3) gives

$$S_{pre} = \begin{pmatrix} 0.4206 & -43.0304 \\ 0.0155 & 0.7946 \end{pmatrix}$$

$$S_{post} = \begin{pmatrix} 0.3404 & -45.9628 \\ 0.0155 & 0.8488 \end{pmatrix}$$

According to Eq. (56.7), we calculate a relative magnification of $M_{pre} = 1.2585$ and $M_{post} = 1.1782$ and a decrease in lateral magnification of 6.4% ($\Delta M = 0.9362$).

Calculation of Toric Additional Lenses

The matrix scheme as outlined before for stigmatic lenses can easily be generalized for toric additional lenses. Instead of 2 × 2, we have to deal with 4 × 4 matrices for the system matrix [28, 30], the refraction, and the translation matrices, which are composed of 4 2 × 2 sub-matrices A, B, C, and D. The slope angles α_x and α_y and the ray height h_x and h_y of the exiting ray in x-direction and y-direction are calculated from the respective slope angles and height values of the incident ray (α_{x0}, α_{y0}, h_{x0} and h_{y0}) by

$$\begin{pmatrix} \alpha_x \\ \alpha_y \\ h_x \\ h_y \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & B_{11} & B_{12} \\ A_{21} & A_{22} & B_{21} & B_{22} \\ C_{11} & C_{12} & D_{11} & D_{12} \\ C_{21} & C_{22} & D_{21} & D_{22} \end{pmatrix} \begin{pmatrix} \alpha_{x0} \\ \alpha_{y0} \\ h_{x0} \\ h_{y0} \end{pmatrix}. \qquad (56.8)$$

The refraction matrix P and the translation matrix T for the astigmatic case are of the form

$$P = \begin{pmatrix} 1 & 0 & -p_a & -p_d \\ 0 & 1 & -p_d & -p_b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{d}{n} & 0 & 1 & 0 \\ 0 & \frac{d}{n} & 0 & 1 \end{pmatrix}, \qquad (56.9)$$

where

$$\begin{aligned} p_a &= p_1 + (p_2 - p_1)\sin 2(ax) \\ p_b &= p_1 + (p_2 - p_1)\cos 2(ax)\ , \\ p_d &= (p_2 - p_1)\sin(ax)\cos(ax) \end{aligned} \qquad (56.10)$$

with the refractive power of a surface in meridian 1 (with radius r_1) $p_1 = (n' - n)/r_1$ and meridian 2 (with radius r_2) $p_2 = (n' - n)/r_2$, and with ax as the orientation of meridian 1.

From the refraction and translation matrices, the 4 × 4 system S_{pre} and S_{post} are calculated according to Eq. (56.3). As S_0 defined in Eq. (56.4) is now a 4 × 4 matrix and A_0, B_0, C_0, and D_0 are 2 × 2 matrices instead of scalars, Eq. (56.5) has to be reformulated to ensure that the vergence at the Add-on plane is identical for the preoperative and the postoperative situation:

$$(B_0 - A_0 p_S) \bullet \text{inv}(D_0 - C_0 P_S) = (B_0 - D_0 p_{\text{Add-on}}) \bullet \text{inv}(D_0). \qquad (56.11)$$

Reformulating Eq. (56.11) yields the refractive power of the Add-on ($p_{\text{Add-on}}$):

$$p_{\text{Add-on}} = \left(B_0 - (\text{inv}(D_0))\left((B_0 - A_0 p_S) \bullet \text{inv}(D_0 - C_0 P_S) \bullet D_0\right)\right), \qquad (56.12)$$

where inv. (.) refers to the inverse of the 2 × 2 matrix (.). Using an eigenvalue decomposition of the 2 × 2 matrix $p_{\text{Add-on}}$ yields the power in both meridians (eigenvalue 1 and 2), and the orientation of meridian 1 is extracted from eigenvector 1.

According to the stigmatic case, if we select the chief ray, which passes through the center of the aperture stop assumed to be located at the Add-on plane, we obtain

$$\begin{pmatrix} h_x \\ h_y \end{pmatrix} = C \begin{pmatrix} \alpha_{x0} \\ \alpha_{y0} \end{pmatrix} + D \begin{pmatrix} h_{x0} \\ h_{y0} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \qquad (56.13)$$

or

$$\begin{pmatrix} h_{x0} \\ h_{y0} \end{pmatrix} = -\text{inv}(D) \bullet C \bullet \begin{pmatrix} \alpha_{x0} \\ \alpha_{y0} \end{pmatrix}. \qquad (56.14)$$

Inserting the result of Eq. (56.14) into Eq. (56.8) yields the lateral magnification for the preoperative or postoperative astigmatic optical system:

$$M_{\text{pre/post}} = A - B \bullet \text{inv}(D) \bullet C. \qquad (56.15)$$

The situations for lateral magnification [31] in both principal meridians before (blue) and after (red) implantation of a toric Add-on are displayed in a sketch in Fig. 56.3. In this example, the meridian of magnification changes from 70° preoperatively to 85° postoperatively, whereas the axis of magnification changes from 160 to 175°. The overall magnification as indicated by the dashed lines increases by 25% from preoperative (blue dashed line) to postoperative (red dashed line).

The relative change in magnification is given by

$$\Delta M = \text{inv}(M_{\text{pre}}) \bullet M_{\text{post}}. \qquad (56.16)$$

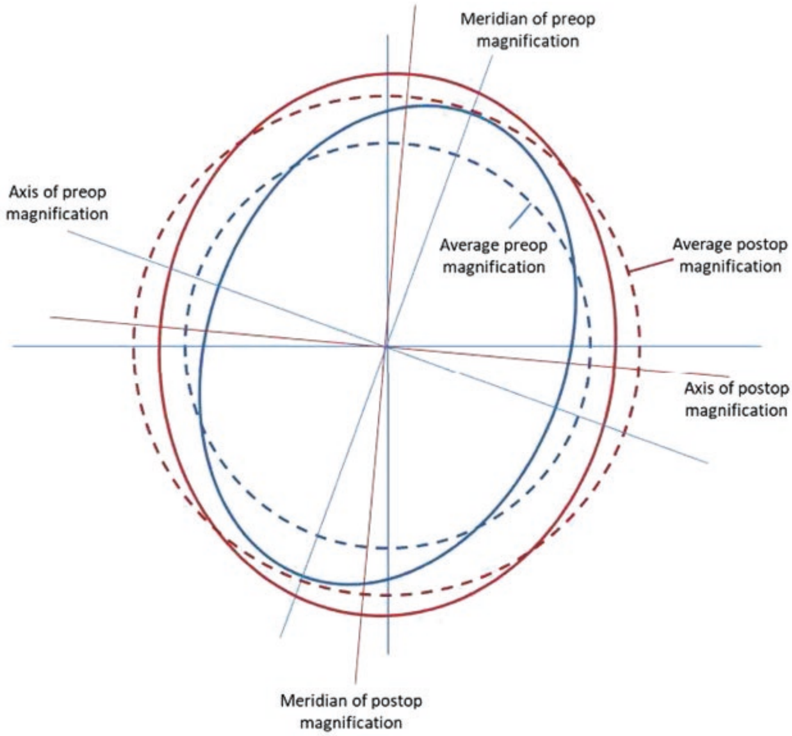

Fig. 56.3 Lateral magnification before (blue) and after (red) implantation of a toric Add-on. In this example, the overall magnification gains by 25% (blue and red dashed circles), which indicates that a minus Add-on is implanted. The image distortion (major to minor axis of the ellipse) is typically reduced if the refraction correction is transferred from the spectacle plane to the Add-on plane

Again, the principal meridians and the orientation of the principal meridians are extracted from M_{pre}, M_{post}, and ΔM using eigenvalue decomposition.

Clinical Example 3

With a phakic lens, preexisting spectacle correction $p_{S1} = -7$ dpt/A = 10° and $p_{S2} = -10$ dpt/A = 100°at a vertex distance VD = 12 mm to be transferred to a correction at ELP = 3.4 mm behind the corneal front apex. With a corneal front surface shape of 8.0 mm/A = 20° and 7.6 mm/A = 110° and a corneal back surface shape of 6.7 mm/A = 25° and 6.4 mm/A = 115° and a central corneal thickness of 500 μm, and refractive indices of air/cornea/aqueous of 1.0/1.376/1.336, S_0 according to Eq. (56.4) reads

$$S_0 = \begin{pmatrix} 0.5057 & 0.0083 & -41.3727 & 0.6915 \\ 0.0083 & 0.4851 & 0.6915 & -43.0951 \\ 0.0138 & 0.0 & 0.8497 & 0.0025 \\ 0.0 & 0.0137 & 0.0025 & 0.8434 \end{pmatrix}$$

The power of the Add-on is derived from Eq. (56.12) as

$$p_{\text{Add-on}} = \begin{pmatrix} 8.8078 & -0.6138 \\ -0.6138 & 11.9972 \end{pmatrix}.$$

Converted to standard notation this gives −8.69/A = 10.53 and −12.11/A = 100.53°, or −12.11 + 3.42/A = 100.53°. According to Eq. (56.3), S_{pre} and S_{post} read

$$S_{pre} = \begin{pmatrix} 0.5057 & 0.0083 & -37.7912 & 0.5139 \\ 0.0083 & 0.4851 & 0.5012 & -38.2921 \\ 0.0138 & 0.0 & 0.9474 & -0.0043 \\ 0.0 & 0.0137 & -0.0043 & 0.9793 \end{pmatrix}$$

$$S_{post} = \begin{pmatrix} 0.5057 & 0.0083 & -41.3727 & 0.6915 \\ 0.0083 & 0.4851 & 0.6915 & -43.0951 \\ 0.0138 & 0.0 & 0.8497 & 0.0025 \\ 0.0 & 0.0137 & 0.0025 & 0.8434 \end{pmatrix}$$

According to Eq. (56.15), we calculate a relative magnification before (M_{pre}) and after (M_{post}) Add-on implantation, and according to Eq. (56.16), the change in relative magnification as

$$M_{pre} = \begin{pmatrix} 1.0556 & 0.0044 \\ 0.0048 & 1.0212 \end{pmatrix}$$
$$M_{post} = \begin{pmatrix} 1.1770 & -0.0035 \\ -0.0035 & 1.1857 \end{pmatrix},$$
$$\Delta M = \begin{pmatrix} 1.1150 & -0.0082 \\ -0.0086 & 1.1611 \end{pmatrix}$$

and using eigenvalue decomposition gives the principal meridians of magnification preoperatively (1.0562/A = 7.8° and 1.0206/A = 97.8°) and postoperatively (1.1757 A = 19.5° and 1.1869/A = 109.5°) together with the gain in ocular magnification from Add-on implantation (11.35% in 10.3° and 16.26% in 100.3°). Lateral image distortion is reduced from 3.49% preoperatively to 0.94% postoperatively. Example 3 demonstrates that the refractive power of an Add-on is determined mostly by the refraction (sphere, cylinder, and axis) and only to a small amount by the cornea (base curve, astigmatism, and axis).

Simplification for a Thin Lens Model of the Cornea

The calculation strategy for Add-on lenses as shown above can be simplified by considering the cornea as a thin lens with a single refractive surface located at the front apex position of the meniscus lens. In general, if the corneal front and back surface data are available and the calculation scheme is computerized, there is no need for this simplification to a thin cornea model. Especially after corneal refractive surgery (e.g., LASIK), it is important to consider both corneal surfaces in the calculation concept to avoid refractive surprises, as the ratio of front-to-back surface curvature of the cornea shows some mismatch. Equation (56.3), which describes the situations of the anterior eye segment from the spectacle plane to the Add-on plane before and after implantation of the Add-on, has to be replaced by

$$\begin{aligned} S_{pre} &= T_{ELP} P_{CK} T_{VD} P_S \\ S_{post} &= P_{Add-on} T_{ELP} P_{CK} T_{VD} \end{aligned}, \quad (56.17)$$

where T_{ELP} refers to the translation matrices for the axial position of the Add-on with respect to the anterior front vertex plane of the cornea, and P_{CK} refers to the refraction matrix describing the keratometric power of the cornea. For the stigmatic case (calculation of non-toric Add-on) and the astigmatic case (calculation of toric Add-on), the matrices P_{CK} and T_{ELP} read

$$\begin{aligned} P_{CK} &= \begin{pmatrix} 1 & -\frac{n_K - 1}{r} \\ 0 & 1 \end{pmatrix} \\ T_{ELP} &= \begin{pmatrix} 1 & 0 \\ \frac{ELP}{n_{Aqueous}} & 1 \end{pmatrix} \end{aligned}, \quad (56.18)$$

and

$$P_{\mathrm{CK}} = \begin{pmatrix} \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} & \begin{matrix} -\left(\frac{n_K-1}{r_1}+\left(\frac{n_K-1}{r_2}-\frac{n_K-1}{r_1}\right)\sin 2(a_1)\right) & -\left(\left(\frac{n_K-1}{r_2}-\frac{n_K-1}{r_1}\right)\sin(a_1)\cos(a_1)\right) \\ -\left(\left(\frac{n_K-1}{r_2}-\frac{n_K-1}{r_1}\right)\sin(a_1)\cos(a_1)\right) & -\left(\frac{n_K-1}{r_1}+\left(\frac{n_K-1}{r_2}-\frac{n_K-1}{r_1}\right)\cos 2(a_1)\right) \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \end{pmatrix},$$

$$T_{\mathrm{ELP}} = \begin{pmatrix} \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} \frac{\mathrm{ELP}}{n_{\mathrm{Aqueous}}} & 0 \\ 0 & \frac{\mathrm{ELP}}{n_{\mathrm{Aqueous}}} \end{matrix} & \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \end{pmatrix} \quad (56.19)$$

Equation (56.4) has to be replaced by

$$S_0 = T_{\mathrm{ELP}} P_{\mathrm{CK}} T_{\mathrm{VD}} = \begin{pmatrix} A_0 & B_0 \\ C_0 & D_0 \end{pmatrix}, \quad (56.20)$$

while all other steps of the calculations remain unchanged.

Simplification Using Linear Modeling

Especially where a computerized calculation scheme is not available in the clinical routine process, the power of a stigmatic Add-on and the change in magnification due to the implantation of an Add-on can easily be estimated using a simple polynomial model. In the case of a toric Add-on, we recommend a calculation instead of such a simplification, as there are some more effect sizes, plus the situation of crossed cylinders which cannot be simplified properly.

As the conversion of refraction from the spectacle plane to the Add-on plane is not linear, we set up a polynomial of third order to describe the effect of $p_{\text{Add-on}}$ and a linear function to model ΔM as a function of the spectacle refraction p_S. All of the other parameters such as corneal front and back surface curvature p_{CA} and p_{CP}, corneal thickness CCT, and the axial position of the Add-on ELP were analyzed and can be linearized with a sufficient clinical precision. We derived the coefficients of the polynomial fit function $p_{\text{Add-on}} = \text{fit}p_{\text{Add-on}}(p_S)$ and ΔM (in %) = $\text{fit}_{\Delta M}(p_S)$ for standard values of p_{CA} = 7.77 mm, p_{CP} = 6.4 mm, CCT = 500 μm and ELP = 3.4 mm for the phakic Add-on using a least squares optimization process:

$$\begin{aligned} p_{\text{Add-on}} &= 2.79\exp{-4} \bullet p_S^3 + 1.88\exp{-2} \bullet p_S^2 + 1.26 \bullet p_S - 2.31\exp{-4} \\ \Delta M(\%) &= -1.49 \bullet p_S \end{aligned}, \quad (56.21)$$

The effect of all other parameters was analyzed by calculating the gradient of $p_{\text{Add-on}} - \text{fit}p_{\text{Add-on}}(p_S)$ and ΔM (in %) − $\text{fit}_{\Delta M}(p_S)$.

For the situation of a phakic Add-on, the power of an Add-on and the change in magnification in % due to the implantation of a stigmatic Add-on can be estimated from the following equation:

$$p_{\text{Add-on}}(\text{dpt}) = 2.79 \bullet \exp{-4} \bullet p_S(\text{dpt})3 + 1.88 \bullet \exp{-2} \bullet p_S(\text{dpt})2 + 1.26 \bullet p_S(\text{dpt})$$
$$-2.31\exp{-4} - 4.55 \bullet \exp{-2}\bullet(p_{CA}(\text{mm}) - 7.77) + 5.99\exp{-3} \bullet (p_{CP}(\text{mm}) - 6.4)$$
$$+1.15\exp{-5} \bullet (\text{CCT}(\mu m) - 500) + 9.21\exp{-2} \bullet (\text{ELP}(\text{mm}) - 3.4) \quad (56.22)$$

$$\Delta M(\%) = -1.49 \bullet p_S + 5.06\exp{-3}\bullet(p_{CA}(\text{mm}) - 7.77) - 5.80\exp{-4} \bullet (p_{CP}(\text{mm}) - 6.4)$$
$$+2.40\exp{-7} \bullet (\text{CCT}(\mu m) - 500) - 9.43\exp{-2} \bullet (\text{ELP}(\text{mm}) - 3.4). \quad (56.23)$$

For the situation of a pseudophakic Add-on, we derived the coefficients of the polynomial fit function $p_{\text{Add-on}} = \text{fit}p_{\text{Add-on}}(p_S)$ and ΔM (in %) = $\text{fit}_{\Delta M}(p_S)$ for standard values of $p_{CA} = 7.77$ mm, $p_{CP} = 6.4$ mm, CCT = 500 µm, and ELP = 4.8 mm using

$$\begin{aligned} p_{\text{Add-on}} &= 3.71\exp{-4} \bullet p_S^3 + 2.28\exp{-2} \bullet p_S^2 + 1.39 \bullet p_S - 3.35\exp{-4} \\ \Delta M(\%) &= -1.62 \bullet p_S \end{aligned}, \quad (56.24)$$

The effect of all other parameters was analyzed by calculating the gradient of $p_{\text{Add-on}} - \text{fit}p_{\text{Add-on}}(p_S)$ and ΔM (in %) $- \text{fit}_{\Delta M}(p_S)$.

For the situation of a phakic Add-on, the power of an Add-on and the change in magnification in % due to the implantation of a stigmatic Add-on can be estimated from the following equation:

$$p_{\text{Add-on}}(\text{dpt}) = 3.71 \bullet \exp{-4} \bullet p_S(\text{dpt})3 + 2.28 \bullet \exp{-2} \bullet p_S(\text{dpt})2 + 1.39 \bullet p_S(\text{dpt})$$
$$-3.35\exp{-4} - 7.51 \bullet \exp{-2}\bullet(p_{CA}(\text{mm}) - 7.77) + 1.04\exp{-2} \bullet (p_{CP}(\text{mm}) - 6.4)$$
$$+1.43\exp{-4} \bullet (\text{CCT}(\mu m) - 500) + 1.08\exp{-1} \bullet (\text{ELP}(\text{mm}) - 4.8) \quad (56.25)$$

$$\Delta M(\%) = -1.62 \bullet p_S + 1.12\exp{-2}\bullet(p_{CA}(\text{mm}) - 7.77) - 1.41\exp{-3} \bullet (p_{CP}(\text{mm}) - 6.4)$$
$$-1.08\exp{-5} \bullet (\text{CCT}(\mu m) - 500) - 1.04\exp{-1} \bullet (\text{ELP}(\text{mm}) - 4.8). \quad (56.26)$$

Figure 56.4 displays the power of an Add-on and the change in magnification if the refractive correction at spectacle plane is converted to Add-on plane for an example with a vertex distance of 12 mm, a corneal front surface/back surface curvature of 7.77/6.4 mm, a corneal thickness of 500 µm, and a ELP of 3.4 mm (for the phakic Add-on) and 4.8 mm (for the pseudophakic Add-on). For a myopic correction ($p_S < 0$), the ratio of $p_{\text{Add-on}}/p_S$ yields lower values compared to a hyperopic correction, which is considered with the polynomial fit function of order 3. The change in magnification can be described using a linear fit function as shown in Eqs. (56.21) and (56.22).

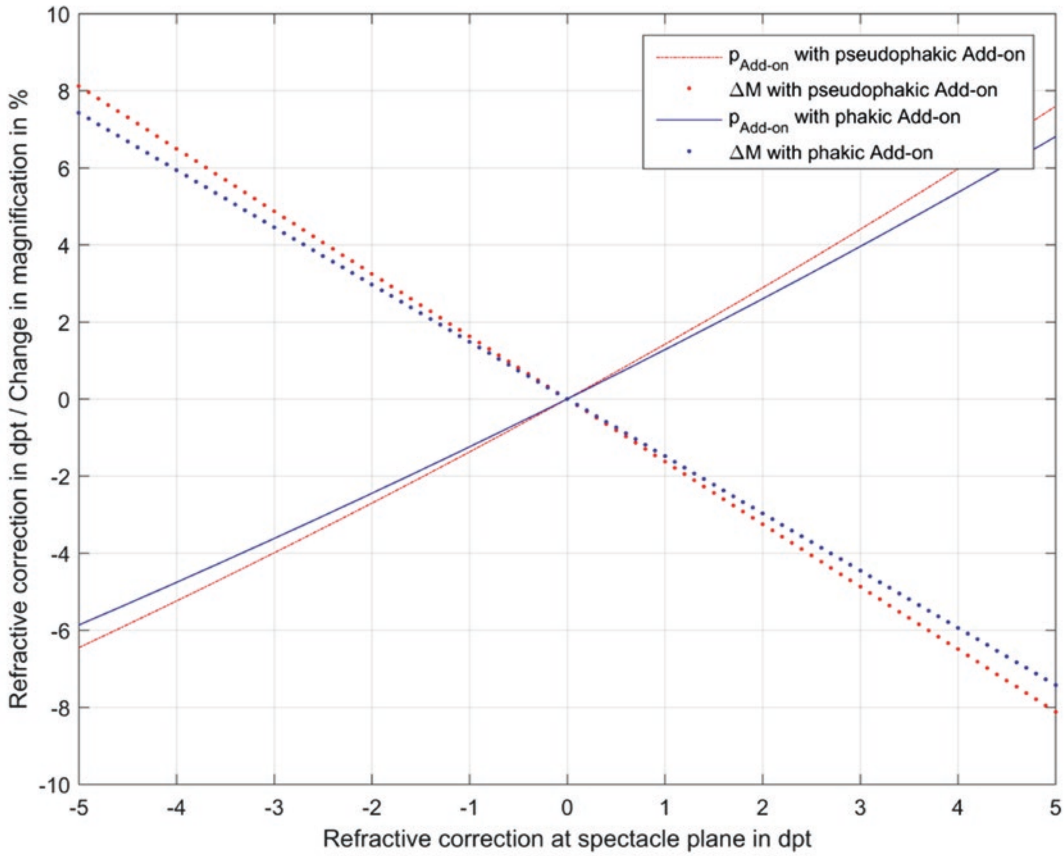

Fig. 56.4 Power of an Add-on and the change in magnification as a function of spectacle refraction to be corrected with the Add-on. This graph depicts an example with a vertex distance of 12 mm, a corneal front surface/back surface curvature of 7.77/6.4 mm, a corneal thickness of 500 µm, and a ELP of 3.4 mm (for the phakic Add-on) and 4.8 mm (for the pseudophakic Add-on)

References

1. Schrecker J, Feith A, Langenbucher A. Comparison of additional pseudophakic multifocal lenses and multifocal intraocular lenses in the capsular bag. Br J Ophthalmol. 2014;98:915–9. https://doi.org/10.1136/bjophthalmol-2013-304591.
2. Gonvers M, Bornet C, Othenin-Girard P. Implantable contact lens for moderate to high myopia: relationship of vaulting to cataract formation. J Cataract Refract Surg. 2003;29:918–24. https://doi.org/10.1016/s0886-3350(03)00065-8.
3. Kohnen T, Kasper T, Bühren J, Fechner PU. Ten-year follow-up of a ciliary sulcus-fixated silicone phakic posterior chamber intraocular lens. J Cataract Refract Surg. 2004;30:2431–4. https://doi.org/10.1016/j.jcrs.2004.04.066.
4. Tsiklis NS, Kymionis GD, Karp CL, Naoumidi T, Pallikaris AL. Nine-year follow up of a posterior chamber phakic IOL in one eye and LASIK in the fellow eye of the same patient. J Refract Surg. 2007;23:935–7.
5. Lackner B, Pieh S, Schmidinger G, Simader C, Franz C, Dejaco-Ruhswurm I, et al. Long-term results of implantation of phakic posterior chamber intraocular lenses. J Cataract Refract Surg. 2004;30:2269–76. https://doi.org/10.1016/j.jcrs.2004.07.018.
6. Boutillier G, Gueudry J, Aouidid S, Muraine M. Sudden rotation and technique for repositioning Add-On® piggy-back sulcus toric intraocular lenses. J Fr Ophtalmol. 2021;44:e287–90. https://doi.org/10.1016/j.jfo.2020.07.015.
7. Muñoz G, Albarrán-Diego C, Belda L, Rohrweck S. Add-on sulcus-based versus primary in-the-bag multifocal intraocular lens: intraindividual study.

J Refract Surg. 2014;30(5):320–5. https://doi.org/10.3928/1081597X-20140422-02.
8. Sauder G. Sekundäre torische intraokularlinsen-implantation in pseudophake Augen. Das "Add-on"-IOL-system [Secondary toric intraocular lens implantation in pseudophakic eyes. The Add-on IOL system]. Ophthalmologe. 2007;104:1041–5. https://doi.org/10.1007/s00347-007-1660-4.
9. Basarir B, Kaya V, Altan C, Karakus S, Pinarci EY, Demirok A. The use of a supplemental sulcus fixated IOL (HumanOptics Add-On IOL) to correct pseudophakic refractive errors. Eur J Ophthalmol. 2012;22(6):898–903. https://doi.org/10.5301/ejo.5000156.
10. Gundersen KG, Potvin R. Refractive and visual outcomes after implantation of a secondary toric sulcus intraocular lenses. Clin Ophthalmol. 2020;18(14):1337–42. https://doi.org/10.2147/OPTH.S255725.
11. Hassenstein A, Niemeck F, Giannakakis K, Klemm M. Torische AddOn-Intraokularlinsen zur Korrektur hoher Astigmatismen nach pseudophaker Keratoplastik [Toric Add-on intraocular lenses for correction of high astigmatism after pseudophakic keratoplasty]. Ophthalmologe. 2017;114(6):549–55. https://doi.org/10.1007/s00347-016-0386-6.
12. Hengerer FH, Conrad-Hengerer I. Pseudophake additive Intraokularlinsen [Pseudophakic additive lenses]. Klin Monatsbl Augenheilkd. 2017;234(12):e43–55. https://doi.org/10.1055/s-0043-120095.
13. Thomas BC, Auffarth GU, Reiter J, Holzer MP, Rabsilber TM. Implantation of three-piece silicone toric additive IOLs in challenging clinical cases with high astigmatism. J Refract Surg. 2013;29(3):187–93. https://doi.org/10.3928/1081597X-20130212-01.
14. Kohnen T, Klaproth OK. Pseudophake additive Intraokularlinsen [Pseudophakic supplementary intraocular lenses]. Ophthalmologe. 2010;107:766–72. https://doi.org/10.1007/s00347-010-2219-3.
15. Schrecker J, Kroeber S, Eppig T, Langenbucher A. Additional multifocal sulcus-based intraocular lens: alternative to multifocal intraocular lens in the capsular bag. J Cataract Refract Surg. 2013;39:548–55. https://doi.org/10.1016/j.jcrs.2012.10.047.
16. Albayrak S, Comba ÖB, Karakaya M. Visual performance and patient satisfaction following the implantation of a novel trifocal supplementary intraocular lens. Eur J Ophthalmol. 2020;6:1120672120969042. https://doi.org/10.1177/1120672120969042.
17. Gekeler K, Gekeler F. Multifokale und Add-on-Intraokularlinsen [Multifocal and Add-on intraocular lenses]. Klin Monbl Augenheilkd. 2014;231(10):1037, 1039–48; quiz 1049-50. https://doi.org/10.1055/s-0033-1358020.
18. Gerten G, Kermani O, Schmiedt K, Farvili E, Foerster A, Oberheide U. Dual intraocular lens implantation: monofocal lens in the bag and additional diffractive multifocal lens in the sulcus. J Cataract Refract Surg. 2009;35(12):2136–43. https://doi.org/10.1016/j.jcrs.2009.07.014.
19. Palomino-Bautista C, Sánchez-Jean R, Carmona Gonzalez D, Romero Domínguez M, Castillo Gómez A. Spectacle Independence for Pseudophakic patients—experience with a trifocal supplementary add-on intraocular lens. Clin Ophthalmol. 2020;14:1043–54. https://doi.org/10.2147/OPTH.S238553.
20. Schrecker J, Langenbucher A. Visual performance in the long term with secondary Add-on versus primary capsular bag multifocal intraocular lenses. J Refract Surg. 2016;32(11):742–7. https://doi.org/10.3928/1081597X-20160630-02.
21. Amon M, Kahraman G, Schrittwieser H. Primäre (Duett Implantation) und sekundäre Implantation additiver Intraokularlinsen bei kindlicher Katarakt. Spektrum Augenheilkd. 2012;26:21–3.
22. Langenbucher A, Viestenz A, Szentmáry N, Seitz B, Viestenz A. Pseudophake und phake torische Linsen zur Korrektur des kornealen Astigmatismus—theorie und klinische Aspekte [Calculation of pseudophakic and phakic toric lenses for correction of corneal astigmatism—theory and clinical aspects]. Klin Monatsbl Augenheilkd. 2008;225(6):541–7. https://doi.org/10.1055/s-2008-1027502.
23. van der Heijde GL, Fechner PU, Worst JGF. Optische Konsequenzen der implantation einer negativen Intraokularlinse bei myopen Patienten. [Optical consequences of implantation of a negative intraocular lens in myopic patients]. Klin Monatsbl Augenheilkd. 1988;193:99–102. https://doi.org/10.1055/s-2008-1050231.
24. Langenbucher A, Szentmáry N, Seitz B. Calculating the power of toric phakic intraocular lenses. Ophthalmic Physiol Opt. 2007;27(4):373–80. https://doi.org/10.1111/j.1475-1313.2007.00487.x.
25. Eppig T, Gillner M, Walter S, Viestenz A, Langenbucher A. Berechnung phaker Intraokularlinsen [Calculation of phakic intraocular lenses]. Klin Monatsbl Augenheilkd. 2011;228(8):690–7. https://doi.org/10.1055/s-0031-1281598.
26. Eppig T, Viestenz A, Seitz B, Langenbucher A. Berechnung pseudophaker torischer Intraokularlinsen [Calculation of pseudophakic toric intraocular lenses]. Klin Monatsbl Augenheilkd. 2011;228(8):681–9. https://doi.org/10.1055/s-0029-1246046.
27. Holladay JT. Refractive power calculations for intraocular lenses in the phakic eye. Am J Ophthalmol.

1993;116(1):63–6. https://doi.org/10.1016/s0002-9394(14)71745-3.
28. Langenbucher A, Reese S, Sauer T, Seitz B. Matrix-based calculation scheme for toric intraocular lenses. Ophthalmic Physiol Opt. 2004;24(6):511–9. https://doi.org/10.1111/j.1475-1313.2004.00231.x.
29. Langenbucher A, Seitz B. Computerized calculation scheme for bitoric eikonic intraocular lenses. Ophthalmic Physiol Opt. 2003;23(3):213–20. https://doi.org/10.1046/j.1475-1313.2003.00109.x.
30. Langenbucher A, Viestenz A, Szentmáry N, Behrens-Baumann W, Viestenz A. Toric intraocular lenses—theory, matrix calculations, and clinical practice. J Refract Surg. 2009;25(7):611–22. https://doi.org/10.3928/1081597X-20090610-07.
31. Langenbucher A, Viestenz A, Seitz B, Brünner H. Computerized calculation scheme for retinal image size after implantation of toric intraocular lenses. Acta Ophthalmol Scand. 2007;85(1):92–8. https://doi.org/10.1111/j.1600-0420.2006.00721.x.