58 Power Calculations for Spherical and Toric Phakic IOLs

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Introduction

A phakic intraocular lens (PIOL) is implanted into the anterior or posterior chamber of a phakic eye to correct hyperopia or myopia with or without astigmatism. PIOLs are specifically designed to be placed in one of three locations: the angle of the anterior chamber, fixated to the iris, or placed in the sulcus. These lenses are available in high powers and are known to provide highly predictable results [1, 2]. Reporting on ten studies (391 total eyes with sphere range from −6.5 to −33 D), the percentage of eyes with postoperative (PO) refraction within ±1.0 D ranged from 63 to 86% with the median being 75% [2].

Several methods have been developed to accurately calculate the power of a PIOL for a given eye. Both stigmatic and astigmatic power calculation methods are described below. The following is intended to be a sampling of these methods.

Stigmatic PIOL Power Calculations

Russian Method

The Russian method [3] is a particularly simple method that calculates the effective spectacle correction (S) referred to the cornea plane where the back vertex distance (bvd) is assumed to be 12.0 mm and the spectacle lens is translated in air. Equation 58.1 provides the power, P_{PIOL}, of the PIOL as derived via the equation for the effectivity of a lens of power P translated a distance d in a medium of refractive index n and then substituted for the PIOL scenario.

$$P_{PIOL} = \frac{P}{1 - \frac{d\,P}{n}} = \frac{S}{1 - \frac{bvd\,S}{1000}} \tag{58.1}$$

The units employed with the effectivity Eq. (58.1) may be chosen from two sets with either optical powers in diopters (D), distances in meters (m), and index of refraction as a value between 1 and 2, OR optical powers in D, distances in millimeters (mm), and index of refraction multiplied by a factor of 1000. The latter system will be used for all equations in this chapter. Note that for positive translation (light propagation toward the retina) d is positive and for negative translation (light propagation out of the eye) d is negative. For example, if S = −10 D, then

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$$P_{PIOL} = \frac{-10.0}{1 - \frac{12.0 \times (-10.0)}{1000}} = -8.93\,\text{D}$$

van der Heijde's Method

A well-established method for calculating the PIOL power is the van der Heijde equation [3, 4]. This method adds the use of the average keratometry power of the cornea (*K*) and effective lens position with respect to the anterior cornea (*elp*). The van der Heijde equation is given in Eq. (58.2).

$$P_{PIOL} = \frac{1336}{\frac{1336}{K + S_c} - elp} - \frac{1336}{\frac{1336}{K} - elp} \quad (58.2)$$

where S_c is the equivalent power for the spectacle lens of power *S* for a given *bvd* as calculated using Eq. (58.1).

Holladay's Method

The vergence formulas of Holladay [5] were derived using a step-along method for the vergence entering the spectacle and eye optical system and stepping along each refracting element (spectacle, cornea, PIOL) and each translation (*bvd* and *elp*). Both the preoperative refraction, *PreRx*, and desired PO refraction, *DPostRx*, are considered. Then, given a selected IOL power (*IOL*), the predicted PO refraction (*PPostRx*) can be calculated. Additionally, a means to optimize the *elp* was provided so the power calculation could be tuned to a surgeon's data set of eyes. The *elp* is calculated using Eq. (58.3).

$$elp = 3.74 + sf \quad (58.3)$$

where *sf* is the surgeon factor optimization parameter and 3.74 is the so-called anatomic anterior chamber depth value [6]. Equation (58.4) calculates the theoretical power for the PIOL.

$$P_{PIOL} = \frac{1336}{\frac{1336}{\frac{1000}{\frac{1000}{PreRx} - bvd} + K} - elp} - \frac{1336}{\frac{1336}{\frac{1000}{\frac{1000}{DPostRx} - bvd} + K} - elp} \quad (58.4)$$

The predicted PO refraction *PPostRx* is calculated using Eq. (58.5).

$$PPostRx = \frac{1000}{\frac{1000}{\frac{1336}{\frac{1336}{\frac{1336}{\frac{1336}{\frac{1000}{\frac{1000}{PreRx} - bvd} + K} - elp} - IOL} + elp} - K} + bvd} \quad (58.5)$$

The surgeon factor is an optimization parameter that can be used to remove bias in the predicted PO refraction for a given data set. For a given PIOL calculation case, the personalized surgeon factor, *psf*, can be calculated to arrive at the "perfect" *sf* value to predict the actual PO refraction (*APostRx*). This is accomplished using Eqs. (58.6)–(58.10).

$$X = \frac{1336}{\frac{1000}{\frac{1000}{PreRx} - bvd} + K} \quad (58.6)$$

$$Y = \frac{1336}{\frac{1000}{\frac{1000}{APostRx} - bvd} + K} \quad (58.7)$$

$$A = IOL \quad (58.8a)$$

$$B = -IOL \times (X + Y) \quad (58.8b)$$

$$C = 1336 \times (X - Y) + IOL \times X \times Y \quad (58.8c)$$

$$elp = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad (58.9)$$

$$psf = elp - 3.74 \quad (58.10)$$

To apply this optimization to a surgeon's data set, Eq. (58.11) is used where *M* is the number of cases in the surgeon's data set. The *psf* is calculated for each case and then the average is taken to yield the new *sf* to be used in Eq. (58.3) for the *elp* value used in Eqs. (58.4) and (58.5).

$$sf = \frac{1}{M}\sum_{m=1}^{M} psf_m \quad (58.11)$$

Astigmatic PIOL Power Calculations

Sarver's Method

Sarver's method [7] combines the step-along method of Holladay with the use of astigmatic decomposition [8] to yield equations to calculate the ideal toric PIOL (TPIOL) power, to predict the PO refraction for a selected TPIOL, and then calculate a simple optimization parameter to remove bias in the predicted PO refraction error spherical equivalent for a given data set. This method makes use of astigmatic decomposition to generalize vergence from stigmatic to astigmatic in a domain where the components may be linearly combined.

Astigmatic Decomposition

The forward astigmatic decomposition transformation is given in Eq. (58.12).

$$m = s + \frac{c}{2}$$

$$c_0 = c \times \cos(2\theta)$$

$$c_{45} = c \times \sin(2\theta) \quad (58.12)$$

where *s, c, and* θ are the standard toric lens parameters; sphere (D), cylinder (D), axis (deg), and *m*, c_0, *and* c_{45} are the astigmatic decomposition parameters (all in diopters) where *m* is the spherical equivalent power.

The axis value θ is the meridian of the toric lens and is limited to the range of 0–180°. For an astigmatic power (in power and axis form) such as keratometric values *steep power @ steep axis + flat power @ flat axis*, the first step is to convert to equivalent s and c axes with negative cylinder values using Eq. (58.13) and then apply Eq. (58.12) to get the equivalent astigmatic decomposition values.

$$s = \text{steep power}$$

$$c = \text{flat power} - \text{steep power}$$

$$\theta = \text{steep axis} \quad (58.13)$$

The inverse astigmatic decomposition transformation for minus cylinder notation is shown in Eq. (58.14).

$$c = -\sqrt{c_0^2 + c_{45}^2}$$

$$s = m - \frac{c}{2}$$

$$\theta = \tan^{-1}\left(\frac{c - c_0}{c_{45}}\right) \quad (58.14)$$

To keep the axis value θ in the range of 0–180, if the calculated value from the arctangent function in Eq. (58.14) is negative, 180 is added to it, and if it is greater than 180, 180 is subtracted from it. Also, note that in programming the astigmatic decomposition equations it is often simpler to use the atan2 function that automatically handles the case where c_{45} is zero.

It is often convenient to denote the forward and inverse astigmatic decomposition transformations of Eqs. (58.12) and (58.14) as shown in Eqs. (58.15) and (58.16), respectively.

$$\begin{bmatrix} m \\ c_0 \\ c_{45} \end{bmatrix} = A\left\{\begin{bmatrix} s \\ c \\ \theta \end{bmatrix}\right\} \tag{58.15}$$

$$\begin{bmatrix} s \\ c \\ \theta \end{bmatrix} = A^{-1}\left\{\begin{bmatrix} m \\ c_0 \\ c_{45} \end{bmatrix}\right\} \tag{58.16}$$

Effectivity

In Eq. (58.1), the effectivity equation for a stigmatic lens or vergence is provided. Its equivalent formulation is given in Eq. (58.17).

$$e_{d,n}(P) = \frac{P}{1-\frac{dP}{n}} = \frac{n}{\frac{n}{P}-d} \tag{58.17}$$

The left-hand side of Eq. (58.17) is intended to provide a shorthand notation for calculating the effectivity of translating a scalar power P a distance d through a medium of index n. The formulation on the right side of the equation is problematic when the power P is zero, so the center formulation is preferred. The structure of the right side is what leads to the characteristic waterfall structure of Eqs. (58.1), (58.2), and (58.4)–(58.7).

To apply the scalar version of the effectivity equation in 17 to an astigmatic decomposition vector or a sphere, cylinder, and axis vector, it is necessary to find the principal powers, translate each one independently, and then transform the results back to either sphere, cylinder, or axis form or to an astigmatic decomposition vector. For an astigmatic decomposition vector **V**, these operations are denoted as shown in Eq. (58.18).

$$E_{d,n}\{\mathbf{V}\} = A\left\{e_{d,n}\left(A^{-1}\{\mathbf{V}\}\right)\right\} \tag{58.18}$$

Phakic IOL Calculations

The parameters used in the TPIOL calculations are either vectors shown with **bold capital letters** (power or vergence values) or scalars shown with *lowercase italics letters* (distance values). They are listed in Table 58.1.

The *elp* value is the *acd* value offset by the *sf* value and is given in Eq. (58.19).

$$elp = acd + sf \tag{58.19}$$

In this section, calculations are presented for ideal IOL power, predicted refraction, back-calculated surgeon factor, and exchange TPIOL power.

Ideal IOL Power

The combination of the preoperative spectacle correction referred to the cornea, the preoperative

Table 58.1 Parameters for the toric phakic IOL calculations

Symbol	Meaning
bvd	Back vertex distance
elp	Expected lens position
aelp	Actual expected lens position
sf	Surgeon factor
acd	Anterior chamber depth (with respect to anterior cornea)
S	Preoperative spectacle power
CL	Preoperative contact lens
K	Preoperative corneal power
PK	PO corneal power
DS	Desired PO spectacle power
PS	Predicted PO spectacle power
AS	Actual PO spectacle power
IP	Ideal power of the TPIOL
AP	Actual implanted IOL power
CP	Current power = AP at PO axis
EP	Exchange TPIOL power

contact lens (if any), and the corneal power all referred to the IOL plane is denoted $\mathbf{P_1}$ and is given in Eq. (58.20).

$$\mathbf{P}_1 = E_{elp,1336}\left\{\mathbf{K} + \mathbf{CL} + E_{bvd,1000}\left\{\mathbf{S}\right\}\right\} \quad (58.20)$$

The combination of the desired PO spectacle correction referred to the cornea and the corneal power referred to the IOL plane is denoted $\mathbf{P_2}$ and is given in Eq. (58.21).

$$\mathbf{P}_2 = E_{elp,1336}\left\{\mathbf{K} + E_{bvd,1000}\left\{\mathbf{DS}\right\}\right\} \quad (58.21)$$

The ideal power of the TPIOL can now be calculated as the difference between the power required at the IOL plane to focus a distant object on the retina $\mathbf{P_1}$ and the power supplied by the desired PO spectacle lens and the cornea referred to the IOL plane $\mathbf{P_2}$. This is specified in Eq. (58.22).

$$\mathbf{IP} = \mathbf{P}_1 - \mathbf{P}_2 \quad (58.22)$$

Predicted PO Refraction

Usually, the precise TPIOL power calculated in eq. 22 will not be available to the surgeon. TPIOL powers are often quantized in step sizes of 0.50 D. For the actual IOL power, **AP,** selected by the surgeon, the PO refraction **PS** can be predicted using Eq. (58.23).

$$\mathbf{PS} = E_{-bvd,1000}\left\{E_{-elp,1336}\left\{\mathbf{P}_1 - \mathbf{AP}\right\} - \mathbf{K}\right\} \quad (58.23)$$

Calculating the *elp* Value

Following Holladay's lead, the actual *elp, aelp,* for each case in a surgeon's data set can be calculated and used to optimize future calculations. For this calculation, instead of using astigmatic decomposition values, it is converted to spherical equivalent values. The calculation is performed by solving a quadratic equation as shown in Eqs. (58.24)–(58.28).

$$a_1 = 1336\frac{bvd\,S - 1000}{(K + CL)\,bvd\,S - 1000(K + CL + S)} \quad (58.24)$$

$$a_2 = 1336\frac{bvd\,AS - 1000}{K\,bvd\,AS - 1000(K + AS)} \quad (58.25)$$

$$b = a_1 + a_2 \quad (58.26)$$

$$c = \frac{1336(a_1 - a_2)}{AP} + a_1 a_2 \quad (58.27)$$

$$aelp = \begin{cases} \dfrac{b - \sqrt{b^2 - 4\ c}}{2} & \text{if } b > 0 \\[2ex] \dfrac{b + \sqrt{b^2 - 4\ c}}{2} & \text{otherwise} \end{cases} \quad (58.28)$$

where *S, K, CL, AS,* and *AP* are scalar spherical equivalent values not astigmatic decomposition vectors.

Refractive Surprise Exchange Calculation

Although not presented in [7], the astigmatic decomposition notation can be used to illustrate how to calculate an exchange PIOL power. The need for this calculation can occur when, for example, there is a refractive surprise that requires a lens exchange. In this case, Eqs. (58.29) and (58.30) can be used to calculate the exchange PIOL power (**EP**).

$$\mathbf{P}_3 = E_{elp,1336}\left\{\mathbf{PK} + E_{bvd,1000}\left\{\mathbf{AS}\right\}\right\} + \mathbf{AP} \quad (58.29)$$

$$\mathbf{EP} = \mathbf{P}_3 - E_{elp,1336}\left\{\mathbf{PK} + E_{bvd,1000}\left\{\mathbf{DS}\right\}\right\} \quad (58.30)$$

Sample Calculation

A sample TPIOL calculation using eqs. 12 to 30 is presented in this section to make clear how the calculations flow. For this example, the scaler data of back vertex distance *bvd* are 12.0 mm, anterior chamber depth *acd* is 3.55 mm, and surgeon factor *sf* is −0.304 mm. Then, from Eq. (58.18), the expected lens position *elp* is calculated to be 3.25 mm. These scaler parameters are listed in Table 58.2.

The second set of data for this example calculation is the astigmatic powers in normal form and astigmatic decomposition form. These values are given in Table 58.3. In this table, the first column contains the vector parameter symbol, the center column contains the normal astigmatic form values, and the last column contains the equivalent astigmatic decomposition vector elements. For parameters already given by sphere, cylinder, and axis values, Eq. (58.12) is used to calculate the astigmatic decomposition elements. For the corneal power vector **K,** Eq. (58.13) is first used to convert from power and axis format to sphere, cylinder, and axis, and then Eq. (58.12) is used to compute the astigmatic decomposition elements.

Table 58.2 Sample TPIOL calculation scalar parameters

Parameter	Value	Equation
bvd	12.0	
acd	3.55	
sf	−0.304	
elp	3.25	(58.18)

Table 58.3 Sample TPIOL calculation astigmatic parameters in normal and in astigmatic decomposition form

Parameter	Sphere, cylinder, axis	M, C0, C45
S	−6.75 − 1.25 × 160.00	−7.38, −0.96, 0.80
CL	0–0 × 0	0, 0, 0
K	42.20 @ 173.00 + 43.20 @ 83.00	42.70, 0.97, −0.24
DS	−0.5 − 0 × 0	−0.5, 0, 0
AP	−8.00 − 1.00 × 160	−8.50, −0.77, 0.64
AS	0.50–1.25 × 180.00	−0.13, −1.25, −0.00
PK	42.2 @ 169 + 43.5 @ 79	42.85, 1.21, −0.47

Using the *elp* value from Table 58.2, the astigmatic decomposition elements from Table 58.3, and Eqs. (58.20)–(58.22), the ideal TPIOL power is calculated to be

$$\mathbf{P}_1 = \begin{bmatrix} 39.36 \\ 0.19 \\ 0.52 \end{bmatrix}$$

$$\mathbf{P}_2 = \begin{bmatrix} 47.03 \\ 1.20 \\ -0.30 \end{bmatrix}$$

$$\mathbf{IP} = \begin{bmatrix} -7.66 \\ -1.01 \\ 0.82 \end{bmatrix}$$

Converting this astigmatic decomposition, TPIOL power to sphere, cylinder, and axis components using Eq. (58.14) gives

$$\mathbf{IP} = -7.01 - 1.30 \times 160^\circ$$

If this **IP** value is used for the actual power **AP** of the TPIOL to be implanted, the predicted PO spectacle would be the desired spectacle **DS** value, −0.5 D. If the surgeon instead plans to use the **AP** power of −8.00 − 1.00 × 160, then using Eqs. (58.14), (58.20), and (58.23), the predicted PO spectacle **PS** is calculated as (astigmatic decomposition and sphere, cylinder, and axis forms)

$$\mathbf{PS} = \begin{bmatrix} 0.18 \\ -0.20 \\ 0.15 \end{bmatrix} \rightarrow 0.30 - 0.25 \times 162^\circ$$

Now, suppose the actual PO spectacle power is 0.50–1.25 × 170° and the surgeon would like to consider an exchange lens. Using Eqs. (58.14), (58.29), and (58.30), the exchange lens parameters are computed in astigmatic decomposition and sphere, cylinder, and axis forms.

$$\mathbf{P}_3 = \begin{bmatrix} 39.18 \\ -072 \\ 0.59 \end{bmatrix}$$

$$\mathbf{EP} = \begin{bmatrix} -8.03 \\ -2.22 \\ 1.17 \end{bmatrix} \rightarrow -6.78 - 2.51 \times 166^{\circ}$$

For available powers near this exchange lens power, the prediction formulas could again be employed to help the surgeon make a selection to yield the best-expected results for the patient.

Next, given the data for this case, the actual *elp* (*aelp*) can be calculated for use in optimizing future results using Eqs. (58.24)–(58.28).

$$a_1 = 37.19$$

$$a_2 = 31.38$$

$$b = 68.58$$

$$c = 254.08$$

$$aelp = 3.93$$

Using this value of 3.93 instead of 3.25 leads to the predicted PO spectacle spherical equivalent equal to the actual PO spectacle spherical equivalent. A data set consisting of historical cases where each one has an *aelp* value could be used to calculate the best constant value *elp* or allow fitting any of a number of machine learning regression models to the data set to improve future results. In the case of using a machine learning regression model, a new *elp* value would be calculated for each future TPIOL case.

Discussion

The set of equations required to support calculations relating to TPIOLs have been provided. This provides the functions of:

1. Calculation of the ideal TPIOL power for a given eye and desired PO spectacle power
2. Prediction of the PO spectacle power for a selected TPIOL power other than the ideal power
3. Calculation of an exchange TPIOL in the event of an unacceptable refractive surprise
4. Back calculation of the *elp* value on a per-case basis to provide a means of building a data set to optimize the predictability of future cases

The same set of equations also supports stigmatic PIOLs, by simply entering 0 for the appropriate cylinder values.

Although not described above, by allowing the surgeon to enter any sphere, cylinder, and axis for the actual TPIOL power **AP**, the effect of axis rotation can readily be calculated using the predicted refraction equations. This analysis would be useful in deciding whether a simple lens rotation would be useful in the event of an unexpected refractive outcome.

MacKenzie [9] and Langenbucher [10] also discuss astigmatic PIOL power calculations using matrix optics and astigmatic decomposition vectors, respectively. The use of matrix optics is numerically equivalent to the astigmatic decomposition approach. However, these references did not present details on how to update the *elp* value to optimize future calculations.

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